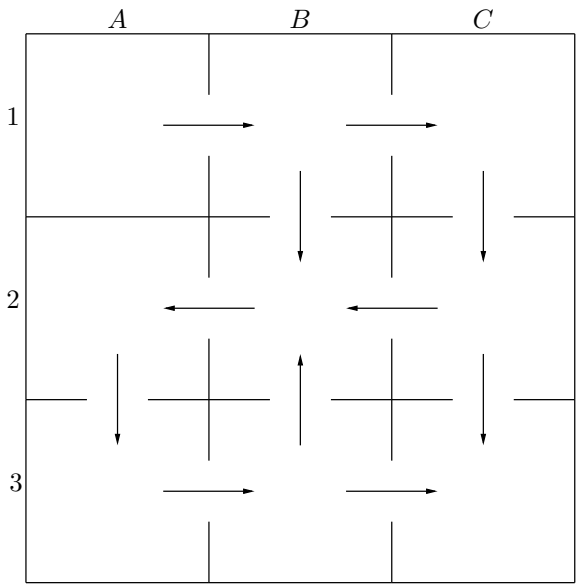
CS3350 Automata, Computability and Formal Languages
Spring 2026. Homework Assignment 2
Due: 04/08/2026 11:59PM MDT
Individual assignment

Provide answers, explanations and proofs for the following questions in the form of a write-up, which, for technical reasons, can be hand-written and scanned.

Submit your write-up in the form of physical paper to your instructor or as a PDF by email to

`utep-spring-2026-automata-hw2@christoph-lauter.org`

1. Consider the following maze made up of one-way-streets:



   The arrows indicate the direction in which movement is possible. A movement from one box into another is indicated with one of the following letters: E (East), W (West), N (North) or S (South). A path through the maze can hence be described as a word over the alphabet $\Sigma = \{E, W, N, S\}$. Let be $L \subseteq \Sigma^\star$ the set of all possible paths from box $A1$ to box $C3$.

   - Give a regular expression describing L.
   - Give a deterministic finite automaton recognizing L.

2. Let be $\Sigma = \{a\}$. Let be $L = \{a^p \mid p \text{ is prime}\} \subseteq \Sigma^\star$. Show that L is not a regular language.

3. Let $M = (Q, \Sigma, \delta, q, F)$ be a deterministic finite automaton. Let $L = L(M) \subseteq \Sigma^\star$ the language accepted by the automaton M. Prove the following statement:

   If there exists a word $w \in L$ with length $|w| \geq \#Q$, where $\#Q$ denotes the number of states in Q, then L contains infinitely many words.

4. Let be $\Sigma = \{a, b\}$. Let be $F \subseteq \Sigma^\star$ the set of all words over Σ that do not contain immediate repetitions of bs[1]. For all $n \geq 0$ let be $F_n = \{w \in F \mid |w| = n\}$, i.e. the set of all words in the language F that are of length n.

[1] This means there can be bs but all occurrences of bs need to be separated by at least one a.

- Give a deterministic finite automaton accepting F.
- Give a regular expression describing F.
- Show that for $n \geq 1$ the cardinality of F_n is the n-th Fibonacci number:

$$\#F_n = \mathrm{fib}(n)$$

where $\mathrm{fib}(n)$ is defined through the recurrence[2]

$$\begin{aligned} \mathrm{fib}(0) &= 1 \\ \mathrm{fib}(1) &= 2 \\ \mathrm{fib}(n) &= \mathrm{fib}(n-1) + \mathrm{fib}(n-2) \quad \forall n \geq 3. \end{aligned}$$

5. Let $\Sigma = \{A, B, C, \ldots Z\}$. Let $L \subseteq \Sigma^\star$ be the language defined by

$$\begin{aligned} L &= \{w \in \Sigma^\star \,|\, w \text{ is a palindrome}\} \\ &= \{w_1 w_2 \ldots w_{n-1} w_n \,|\, n \geq 0,\, w_i \in \Sigma,\, w_i = w_{n-i+1}\, \forall i = 1 \ldots n\}\,. \end{aligned}$$

   Show that L is not regular. Remark: $\varepsilon \in L$.

   Reminder: a palindrome is a word that reads the same backward as forward. Examples include "RADAR", "MADAM" or "RACECAR".

6. Still let Σ be all upper-case, non-accentuated letters in the Latin alphabet. Still let L be the language of all palindromes over this alphabet Σ. Show that L is context-free.

7. Let $M = (\Sigma, \Gamma, Q, \delta, q)$ be a pushdown automaton. Show that there exists a pushdown automaton $M' = (\Sigma, \Gamma', \{q'\}, \delta', q')$ that has only one state q' and recognizes the same language:

$$L(M) = L(M').$$

   You may refer to theorems shown in class or in the textbook without proving them in detail; your argument needs to be explicit, though.

[2] Caution: there are various definitions of the Fibonacci sequence. For this assignment, you must use the one given, i.e. the one for which $\mathrm{fib}(0) = 1$ and $\mathrm{fib}(1) = 2$.