CS3350 Automata
Spring 2026. Midterm I
03/04/2026 – 3:00PM to 4:20PM MST

All documents allowed - Internet access NOT allowed

This midterm exam is open-book, **without** Internet access. All communication with other humans or AIs is prohibited. You need to submit your work on paper directly to your instructor at the end of the exam.

The sum of possible points for this exam is $130 > 100$**. Students receive an A with a score of** ≥ 90 **points.**

Your instructor predicts that there will immediately be some students who think that this exam is too long, too hard, unfair etc. or everything altogether. People! There are 30 *extra points*[1].

1. (20 points) Let $\Sigma = \{a, b, -\}$. Let $L \subseteq \Sigma^\star$ be the language defined by

$$\begin{aligned} L &= \{1 \text{ or more occurrences of the word } abba, \text{separated by hyphens}\} \\ &= \{abba, abba - abba, abba - abba - abba, \dots\}. \end{aligned}$$

   Show that L is regular. Caution: $\varepsilon \notin L$!

2. (23 points) Draw the transition graph of the non-deterministic finite automaton (NFA) $M = (Q, \Sigma, \delta, q_0, F)$ where

$$\begin{aligned} Q &= \{q_0, q_1, q_2, q_3\} \\ \Sigma &= \{a, b\} \\ F &= \{q_3\} \end{aligned}$$

   and the transition function $\delta : Q \times \Sigma_\varepsilon \to \mathcal{P}(Q)$ is given by the following table:

$q \in Q$	$v \in \Sigma_\varepsilon$	$\delta(q, v) \in \mathcal{P}(Q)$
q_0	ε	$\{q_1, q_2\}$
q_1	a	$\{q_1\}$
q_1	b	$\{q_3\}$
q_2	b	$\{q_2, q_3\}$
q_2	a	$\{q_3\}$

   Now proceed by removing the ε-transitions of the automaton M, of course while maintaining its language the same. Finally convert the NFA M to a deterministic finite automaton (DFA) $M' = (Q', \Sigma, \delta', q', F')$, using the powerset construction algorithm, removing all unreachable states in your final answer.

3. (19 points) Provide an NFA $M = (Q, \Sigma, \delta, q_0, F)$ that accepts the regular language described by the regular expression

$$R = x^\star (y|z).$$

   Explain how you proceed when constructing your NFA M.

[1] This means: you loose 40 points and you still get an A.

4. (21 points) Prove the following theorem:
Let Σ be an alphabet. Let $L \subseteq \Sigma^\star$ be a regular language. Then the complement language

$$\overline{L} = \{w \in \Sigma^\star | w \notin L\}$$

is regular.

5. (23 points) Prove the following theorem:
Let Σ be an alphabet. Let $L \subseteq \Sigma^\star$ be a regular language. Then the mirror language

$$M = \{w_{n-1}w_{n-2}\dots w_1 w_0 |\, w_0 w_1 \dots w_{n-2} w_{n-1} \in L,\ \forall i,\ w_i \in \Sigma\}$$

is regular.
The mirror language contains all words of the language but written with the last character first.

6. (24 points) Let $\Sigma = \{a, b, c\}$. Let L be the language defined by

$$L = \left\{a^m b^n c^{m \bmod n} |\, m, n \in \mathbb{N}, 2 \leq n \leq 5\right\}.$$

Is L regular? Provide an answer and a proof for your answer.
Caution: this question is tricky! You may use the fact established by theorem for the question that preceeds.