

**CS5350 Advanced Algorithms
Assignment 2**

Due: 10/02/2023 - 5:59PM MDT

1. The Fibonacci sequence can be defined by $T(1) = 1$, $T(2) = 1$ and

$$T(n) = T(n-1) + T(n-2), \quad n \geq 3.$$

Binet proposed a *closed formula* for the Fibonacci sequence:

$$B(n) = \frac{\left(\frac{1+\sqrt{5}}{2}\right)^n - \left(\frac{1-\sqrt{5}}{2}\right)^n}{\frac{1+\sqrt{5}}{2} - \frac{1-\sqrt{5}}{2}}.$$

Show that Binet's formula is correct, i.e. that

$$B(n) = T(n) \quad \forall n \geq 1.$$

2. The Toom-Cook Multiplication Algorithm splits the two input integers a and b , both of size n , into three parts each

$$\begin{aligned} a &= \beta^{\frac{2n}{3}} a_h + \beta^{\frac{n}{3}} a_m + a_l \\ b &= \beta^{\frac{2n}{3}} b_h + \beta^{\frac{n}{3}} b_m + b_l \end{aligned}$$

combines the six parts a_h through b_l with $\mathcal{O}(n)$ operations, obtaining intermediate values s_1 through s_5 and r_1 through r_5 , each of which has size $\frac{n}{3}$, executes 5 recursive calls to compute five products $t_1 = s_1 \times r_1$ through $t_5 = s_5 \times r_5$ and finally combines these five products t_1 through t_5 in such a way to obtain the complete product $c = a \times b$, using $\mathcal{O}(n)$ operations.

Using the Master Theorem, show that the complexity of the Toom-Cook Multiplication Algorithm is $\mathcal{O}(n^{\log_3 5})$. Determine whether the Toom-Cook or the Karatsuba Algorithm is faster. Determine whether the Toom-Cook or the Schoolbook Multiplication Algorithm is faster.